Numerical and experimental comparison between 2-station and multistation methods for spectral analysis of surface waves

1. Introduction

Many relevant problems in Earthquake Geotechnical Engineering and Soil Dynamics, including site response analysis and design of vibrating machine foundations, need a sufficiently accurate evaluation of the soil stiffness properties. In particular the stiffness at very low strains is a fundamental parameter for every dynamical soil model and its evaluation with laboratory test can be problematic. This aspect is particularly important in hard-to-sample materials such as cohesionless soils, in which obtaining undisturbed samples is difficult and very expensive.

In situ small strain stiffness is often obtained with seismic methods, which are based on wave propagation. Boreholes seismic methods such as the Cross-Hole and the Down-Hole tests are very accurate but costly and time consuming. Non-invasive surface wave based methods offer an acceptable accurateness without the need for boreholes.

The stiffness profile is estimated with an inversion process using the dispersive properties of surface waves. The main advantage is that Rayleigh and Love waves can be detected on the ground surface. Early applications were developed by seismologists for the characterisation of the Earth's crust [DORMAN and EWING, 1962; AKI and RICHARDS, 1980].

On the engineering side, small scale simplified applications were proposed for soil and pavement characterisation using the Steady State Rayleigh method [JONES, 1958].

But the wide spread of surface wave based methods in geotechnical engineering started from the middle Eighties, when the researchers of the University of Texas at Austin developed the SASW (Spectral Analysis of Surface Waves) method. This method combined a rigorous approach for the inversion process with a strong testing time reduction in the field [NAZARIAN and STOKOE, 1984]. Successively several similar techniques have been proposed for soil characterisation, either based on the use of active sources or micro-tremors [see FOTI, 2000].

In its original configuration the SASW method uses an impact source and a couple of receivers connected to a signal analyser [NAZARIAN and STOKOE, 1984]. Such simple test setup has given a strong impulse for the diffusion of the method, but it has some inherent drawbacks, which are related to the necessity of repeating the test in several different configurations and to some difficulties in data interpretation, as it will be explained in Section 2.1.

The use of multistation testing configurations overcomes most of these limitations. Multistation detection and analysis of impulsive signals have been successfully applied for soil characterisation on large-scale projects [GABRIELS *et al.*, 1987; MCMECHAN and YEDLIN, 1981], but they are not commonly used for geotechnical applications.

This paper is focused on the comparison between the multistation approach and the traditional

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a constant velocity, which is function only of the elastic parameters. In heterogeneous elastic media their velocity of propagation is a function of frequency and such dependence is strictly related to the stiffness variation with depth. Hence it is possible with a process of inversion to extract information about the stiffness profile from the relationship between velocity of propagation of Rayleigh waves and frequency.

Such inversion process requires the choice of a reference model for the medium and the obtained results will be strongly dependent on such a choice. A horizontally layered elastic model is usually adopted for soil deposits. The results of a SASW inversion analysis can be considered reliable only if there is a sufficient correspondence of the model to the actual geometry of the soil deposit.

Some techniques based on the acquisition and analysis of microtremors have been proposed [TOKIMATSU, 1995], but typical engineering applications of surface wave methods involve the generation of a wavefield using a source acting on the ground surface. The source can be either impulsive, as for example a sledge-hammer or a weight-drop system, or controlled, able to reproduce a harmonic input in the ground. The advantage of the latter option is the possibility of increasing the signal to noise ratio [RIX, 1988], but it requires more sophisticated and expensive equipment.

The wavefield generated by a source acting on the ground surface is not uniquely composed of Rayleigh waves, but also of body waves, which represent a disturbance. Nevertheless Rayleigh waves typ- face and analysed to extract the experimental dispersion curve, i.e. the relationship between velocity of propagation of surface waves and frequency. A variety of techniques have been proposed for this purpose [see FOTI, 2000]. In the following the two-station procedure of the SASW test and the multistation procedure based on the frequency-wavenumber analysis are discussed.

2.1. The SASW method

The typical configuration for the SASW test is represented in Fig. 1. An impact source creates a wave-train, which has components in a broad frequency range. The ground motion is detected by a pair of receivers, which are placed along a straight line passing from the source, and the signals are then analysed in the frequency domain. The phase velocity V_R is obtained from the phase difference of the signals using the following relationship:

$$V_R(\omega) = \frac{\omega}{\Theta_{12}(\omega)} \cdot X \qquad (1)$$

in which $\Theta_{12}(\omega)$ is the cross-power spectrum phase, ω is the angular frequency and X is the inter-receiver spacing (Fig. 1).

One critical aspect of the above procedure is the influence of signal-to-noise ratio. Indeed the measurement of phase difference is a very delicate task. The necessary check on the signal to noise ratio is usually accomplished using the coherence function [SANTAMARINA and FRATTA, 1998], whose value is and spatial aliasing in the recorded signals. In this respect, usually a filtering criterion (function of the testing setup) is applied to the dispersion data [GANJI *et al.*, 1998]. E.g. only frequencies for which the following relationship is satisfied are retained:

$$\frac{X}{3} < \lambda_R(\omega) < 2D \tag{2}$$

in which $\lambda_R(\omega) = V_R(\omega)/f$ is the estimated wavelength, D is the source-first geophone distance, and X is the inter-receiver spacing (Fig. 1). Typically, the receiver positions are such that X and D are equal, in accordance to the results of some parametric studies about the optimal test configuration [SANCHEZ-SALINERO, 1987].

The above filtering criterion assumes that near fields effects are negligible if the first receiver is placed at least half a wavelength away from the source, for a given frequency in the spectral analysis. Such assumption is acceptable in a normally dispersive site, i.e. a site having stiffness increasing with depth, but it can be optimistic for more complex situations [TOKIMATSU, 1995]. For this reason and in order to avoid great loss of data, inversion methods that take into account near field effects have been proposed [ROESSET *et al.*; 1991, GANJI *et al.*, 1998].

For the aforementioned considerations a single testing configuration gives information only for a particular frequency range, which is dependent on receiver positions. The test is then repeated using a variety of geometrical configurations which include ing configurations is assembled and averaged to estimate the experimental dispersion curve at the site, which will be used for the subsequent inversion process.

A very ticklish task in the interpretation of the SASW test is related to the unwrapping of the Cross-Power Spectrum phase. Indeed it is obtained in a modulo-2π, which is very difficult to interpret and unsuitable for further processing [POGGIAGLIOLMI *et al.*, 1982]. The passage to an unwrapped (full-phase) curve is necessary for the computation of time delay as a function of frequency (see Eq. 1).

Usually some automated algorithms are applied for this task [POGGIAGLIOLMI *et al.*, 1982], but external noise can produce fictitious jumps in the wrapped phase, which drastically damage the results. Not always the operator can correct such unwrapping errors on the basis of judgement and in any case it is a subjective procedure, which precludes the automation of the process. An automated procedure based on a least-square interpolation of the cross-power spectrum phase has also been proposed [NAZARIAN and DESAI, 1993].

2.2. *Multistation approaches*

The use of a multistation testing configuration has some evident advantages because of the greater amount of simultaneously collected information.

Such information can be profitably used for a rapid and stable estimation of the dispersion curve, as will be shown in the following. Moreover using a

$$S_m(\omega, x) = I(\omega) \cdot P_m(\omega) \cdot R_m(\omega) \cdot \frac{e^{-\alpha_m(\omega)x}}{\sqrt{x}} \qquad (4)$$

is a combination of instrument response $I(\omega)$, source spectrum $P_m(\omega)$ and path response $R_m(\omega)$ with geometric (represented by the factor $\frac{1}{\sqrt{x}}$) and material (coefficient α_m) attenuation. Notice that all the above factor are frequency dependent.

The modal wavenumber k_m is inversely proportional to the modal phase velocity V_{Rm}:

$$k_m(\omega) = \frac{\omega}{V_{Rm}(\omega)} \,. \qquad (5)$$

The dependence of S_m on distance from the source is only related to the attenuation phenomenon. The influence of the geometrical attenuation

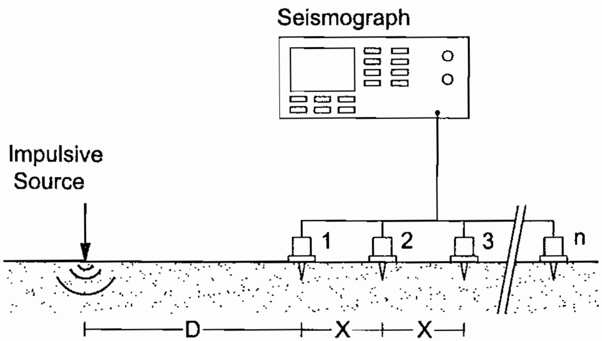

Fig. 2 – Multistation configuration (fk analysis).
Fig. 2 – Configurazione di prova multicanale (analisi fk).

Once the modal wavenumbers have been estimated for each frequency, they can be substituted in Equation 5 to evaluate the modal dispersion curves.

The proof has been outlined considering the displacement spectrum, however in the usual practice velocity transducers (geophones) are used, hence the results are typically reported in terms of velocity spectra.

Application of the *fk* analysis procedure is shown in Fig. 3 using synthetic data. A three-dimensional plot of the velocity spectrum is reported in Fig. 3-a. The contour plot (Fig. 3b) is a more used graphical representation because it shows the location in the *fk* plane of the maxima. The phase velocity of surface waves for a given frequency is evaluated from the location of the maximum in the corresponding "slice" of the spectrum (Fig. 3c). E.g. for the synthetic data shown in Fig. 3 it is possible to obtain.

$$V_{Rm}(50Hz) = \frac{2\pi \cdot f}{k|_{F=\max}} = \frac{2\pi \cdot 50}{0.97} = 324 \quad m/s$$

The process is repeated over the frequency range of interest. In practice the actual range of frequencies is related to the signal-to-noise ratio. Moreover the upper bound is limited by spatial aliasing, that can be easily recognised by the wrap-around of the spectrum [YILMAZ, 1987].

Similarly it can be shown [MCMECHAN and YEDLIN, 1981] that the dispersion curve can be obtained from the spectral maxima in the frequency-slowness

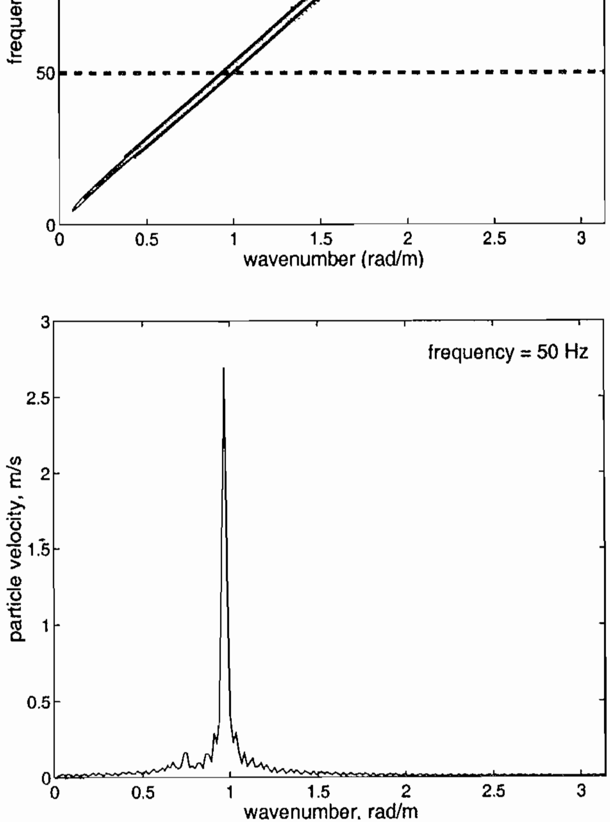

Fig. 3 – Example of fk analysis of surface waves on synthetic data: a) fk spectrum (3D plot) b) fk spectrum (contour plot) c) fk spectrum at frequency=50Hz.

Fig. 3 – Esempio di analisi fk delle onde superficiali su dati sintetici: a) spettro fk (rappresentazione 3D) b) spettro fk (curve di livello) c) spettro fk per frequenza=50Hz.

A comparative numerical analysis has been performed using Rayleigh waves synthetic seismograms, which have been generated using a computer code by R.B. Herrmann and his co-workers of S.Louis University [HERRMANN, 1996]. The impact source has been modelled as an impulsive vertical point source and the corresponding seismograms have been evaluated at a given number of detection points on the ground surface along a straight line passing through the source.

Three profiles have been analysed: Case A represents a normally dispersive medium, with soil stiffness increasing with depth without strong impedance jumps; in Case B a stiff top layer is placed above a normally dispersive medium, generating an inversely dispersive system; finally Case 3 is designed to simulate the presence of a stiff bedrock below a homogenous soil. The above profiles represent typical subsoil conditions that can be found in geotechnical engineering problems.

For the two-station SASW method, 5 receiver configurations have been used considering respectively the following values of the inter-receiver spacing: 1 m, 2 m, 5 m, 10 m and 20 m. The phase velocity has been evaluated using Equation 1 and the information has been filtered according to the criterion of Equation 2.

The multistation *fk* analysis has been applied on a set of 24 synthetic traces with inter-receiver spacing equal to 1m. The use of 24 traces is related to the usual number of channels of common commercial seismographs.

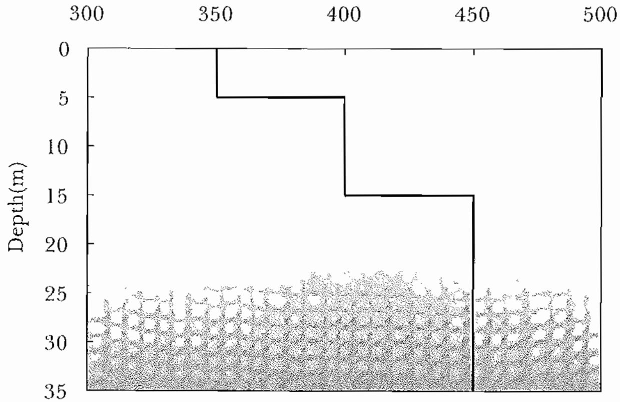

Fig. 4 – Case A: shear wave velocity profile.
Fig 4 – Caso A: profilo di velocità delle onde di taglio.

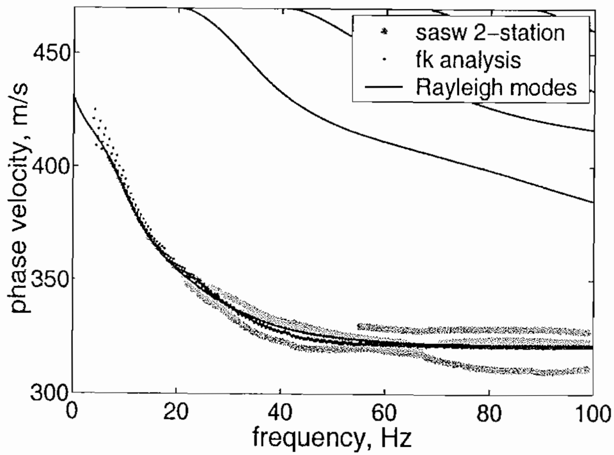

Fig. 5 – Case A: estimated dispersion curve.
Fig. 5 – Caso A: stima della curva di dispersione.

1992; Tokimatsu, 1995] have already shown the strong influence of higher modes for inversely dispersive media. It is important to observe (Fig. 7) that in this case higher mode influence is sensible as frequency increases. Indeed the asymptotic value of the phase velocity for increasing frequency is strongly related to the first layer stiffness. Such asymptotic behaviour can be obtained only with a continuous switch of dominating mode from the fundamental one towards higher ones.

Both the two-station method and the multistation fk analysis yield a dispersion curve, which is determined by mode superposition (Fig. 7). Following the suggestion of Tokimatsu [1995], we will refer to such phase velocity as apparent velocity of Rayleigh waves. It can be shown that, over long distances, the difference of group velocity between modes produces a mode separation effect and, using the fk analysis, it is possible to obtain the dispersion curves of several distinct modes [Gabriels *et al.*, 1987; Foti *et al.*, 2000].

As for case A, also in this case the multistation fk analysis produces a unique estimate in comparison to the sparse values that are obtained using the two station procedure over several receiver pairs.

The implications of the above results on the inversion process must be carefully evaluated. It is important to point out that, independently on the procedure used to estimate the experimental dispersion curve, a fundamental mode approach can not be used. Indeed for an inversely dispersive medium, the higher modes play a strong role in the propagation of surface waves and their influence can not be neglected. Hence for an accurate evaluation of soil

Fig. 6 – Case B: shear wave velocity profile .
Fig. 6 – Caso B: profilo di velocità delle onde di taglio.

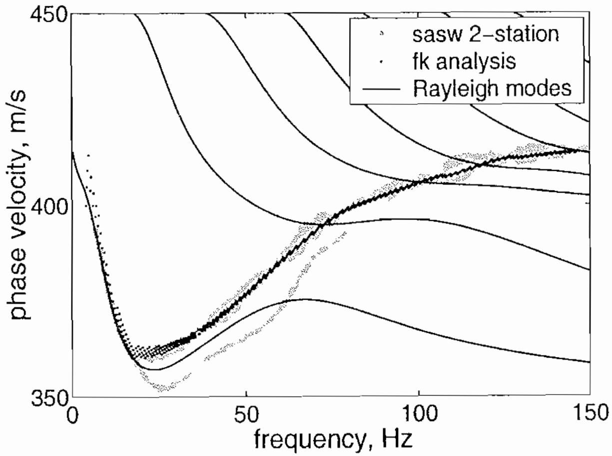

Fig. 7 – Case B: estimated dispersion curve.
Fig. 7 – Caso B: stima della curva di dispersione.

parameters, an inversion process based on a consistent definition of the apparent phase velocity must be used [Lai, 1998].

3.3. Case C

The last synthetic profile represents the typical case of a soil layer over a bedrock (see Tab. III and Fig. 8). In this case the stiffness is increasing with depth and hence the profile is normally dispersive. Nevertheless the strong impedance jump makes

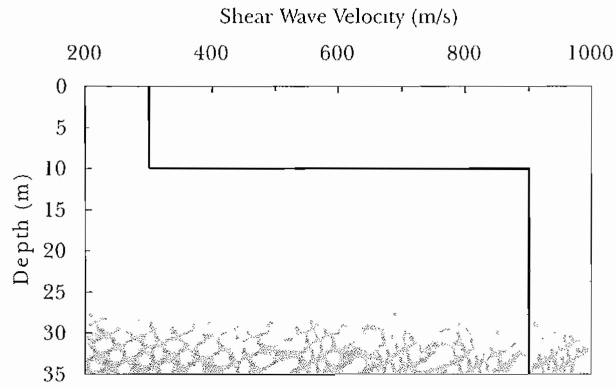

Fig. 8 – Case C: shear wave velocity profile.
Fig. 8 – Caso C: profilo di velocità delle onde di taglio.

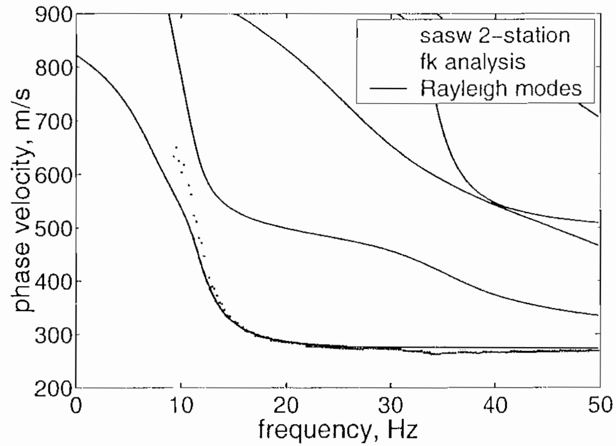

Fig. 9 – Case C: estimated dispersion curve.
Fig. 9 - Caso C: stima della curva di dispersione.

system parameters have a minor influence on the dispersion curve [NAZARIAN, 1984].

Obviously, in the comparison of surface waves and borehole tests results, the inherent differences in volume of tested soil must be kept in mind. Indeed the cross-hole measurement is made between very close points, the surface wave test measures the average properties over a section of several tens of meters and the down-hole method measures average soil properties along depth. This aspect obviously affects the resolution that can be expected from each one of the above methods. Clearly the surface wave method has a lower resolution, e.g. it is not able to detect thin layers at great depth. Nevertheless it has several advantages related to time, cost and possibility of testing the soil without any drilling and casing disturbance.

4.1. Site A

The test site is located in Saluggia (VC) in the northern part of Italy, close to the Dora Baltea River and it is part of a large flat area of fluvial sediments. The soil is composed basically of gravels and gravelly sands, with the presence of fine sand and clayey silt, in the form of lenses. The water table is at very shallow depth, between 2 and 3 meters below the ground surface. The results of a CH test at the site are available from a previous geotechnical survey.

The data have been collected using two different test arrangements having respectively receiver spacing equal to 1 m and 3 m and with the source-

m gather are available, while for high frequencies (above 35Hz) the other gather supplies the information.

The experimental dispersion curve implicitly contains the information relative to the geometry and the mechanical parameters of the soil deposit. An inversion process based on the fundamental mode has been used in this case, because the site is clearly normally dispersive and only the fundamental mode has been recovered from the analysis of the seismic gathers. The starting profile for the inversion process has been selected on the basis of approximate procedures for the estimate of the shear

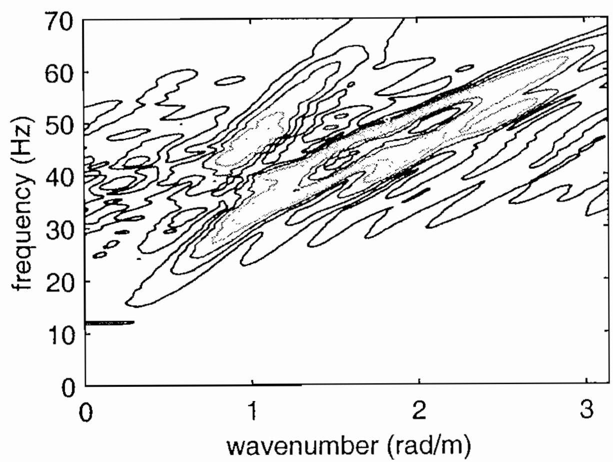

Fig. 10 – Site A: *fk* spectrum (receiver spacing: 1m; source: sledge-hammer).

Fig. 10 – Sito A: spettro fk (spaziatura ricevitori: 1m; sorgente: mazza 6kg).

configuration having a short receiver spacing (0.5 m) has been selected. The source used for this survey was the 6 kg hammer, which was stroke at 1 m and at 5 m from the first receiver. For the two-station procedure the signal pairs at the following inter-receiver distance have been used: 1 m, 2 m, 4 m, 6 m and 8 m.

The presence of the stiff top layer makes this profile inversely dispersive and the obtained experimental dispersion curve (Fig. 14) must be considered as the superposition of several modes of propagation.

In this case a different approach must be used for the inversion process. A numerical estimate of the apparent velocity that is associated to mode superposition can be derived from the numerical analysis of the wavefield generated by a point source at short distances from the source itself. An iterative inversion procedure based on such estimate has been used to obtain the shear wave velocity profile (Fig. 16). The numerical apparent phase velocity corresponding to the profile is compared with the experimental one in Fig. 15.

4.3. Site C

Site C is located in the Tuscany region, in the central part of Italy. The geology of the area is such that the presence of a quite stiff soil was expected at shallow depth below a softer layer (about 10m) [FERRINI, 2000]. A DH survey has been performed after the surface wave test.

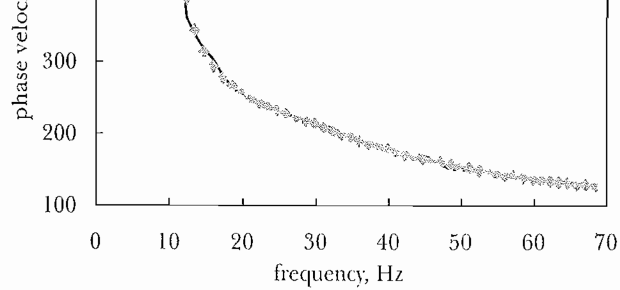

Fig. 12 - Site A: experimental vs. numerical dispersion curve.
Fig. 12 – Sito A: confronto tra curve di dispersione sperimentale e numerica.

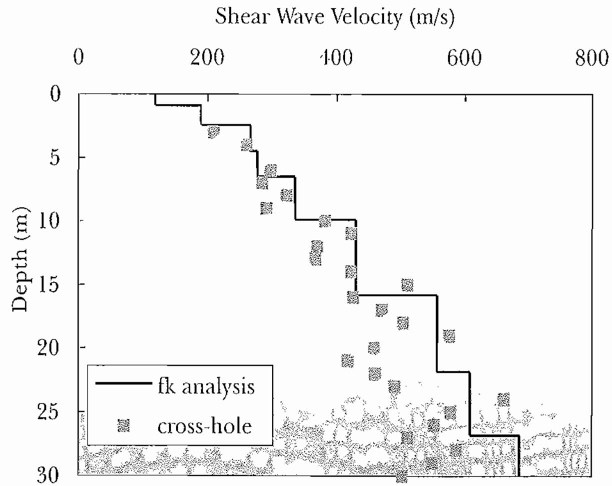

Fig. 13 – Site A: shear wave velocity profile.
Fig. 13 – Sito A. profilo di velocità delle onde di taglio.

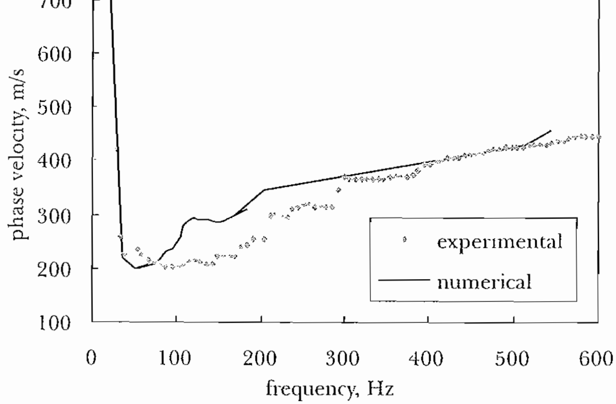

Fig. 15 – Site B: experimental vs. numerical dispersion curve.
Fig. 15 – Sito B: confronto tra curve di dispersione sperimentale e numerica.

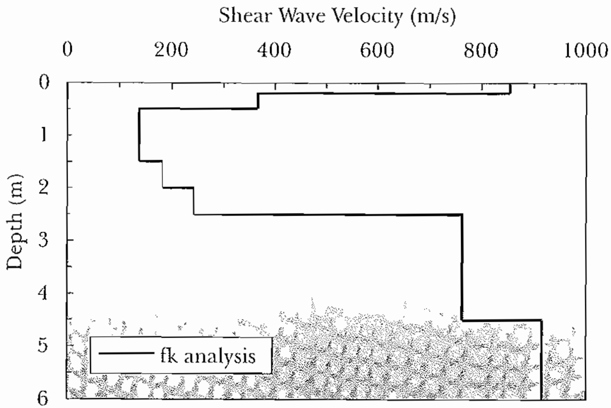

Fig. 16 – Site B: shear wave velocity profile.
Fig. 16 – Sito B: profilo di velocità delle onde di taglio.

Surface waves based methods are increasing their popularity, which is due mainly to the possibility of testing undisturbed soils avoiding the costs related to boreholes. In this paper a wide spectrum comparison between two-station and multistation methods has been presented, considering both synthetic and experimental data. The obtained dispersion curves can be considered practically equivalent for the purposes of the inversion process aimed at estimating the stiffness profile.

Nevertheless the *fk* analysis implies a reduced number of testing and interpretation steps with respect to the two-station method, resulting in a considerable saving of time. The testing time in situ is strongly reduced because only one or two testing configurations are required. Moreover it must be considered that the *fk* analysis is less influenced by external noise, which often prevent two-station data to be correctly interpreted, and, as a consequence, there is much less necessity of stacking experimental signals. For instance it must be considered that the experimental data presented in this paper have been obtained with a single shot for the *fk* analysis, while the traditional two-station interpretation requires a minimum of 3-5 stacks for each testing configuration.

The data processing is much faster and it requires only a very limited operator judgement resulting in a strong automation of the process. This is also due to the fact that *fk* analysis yields directly a single dispersion curve, with no need for averaging.

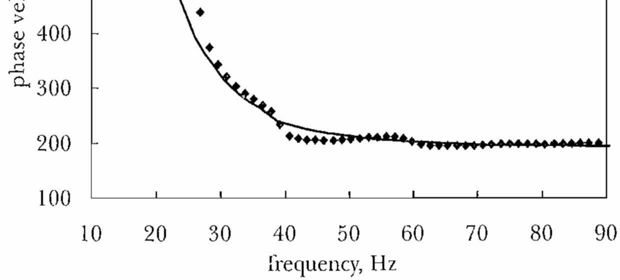

Fig. 18 – Site C: experimental vs. numerical dispersion curve.
Fig. 18 – Sito C: confronto tra curve di dispersione sperimentale e numerica.

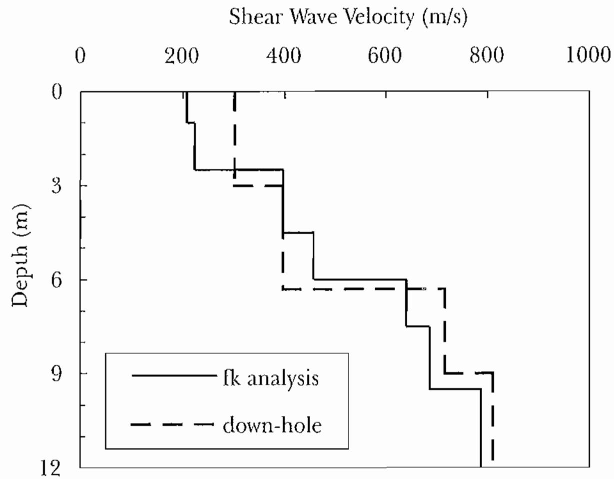

Fig. 19 – Site C: shear wave velocity profile.
Fig. 19 – Sito C: profilo di velocità delle onde di taglio.

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Sommario

I metodi di indagine geofisica basati sulla propagazione delle onde di Rayleigh consentono una ricostruzione sufficientemente dettagliata delle variazioni di rigidezza con la profondità. Essi si stanno recentemente diffondendo nell'ambito della geofisica applicata e dell'ingegneria geotecnica, per effetto dei vantaggi economici ed ingegneristici derivanti dalla possibilità di evitare fori di sondaggio.

Nel metodo SASW, che rappresenta la variante più diffusa in ambito geotecnico di tali metodi, i dati sperimentali vengono collezionati in sito con una configurazione di prova a due ricevitori ed analizzati con una procedura basata sulla differenza di fase tra i segnali.

Nel presente articolo viene presentato un confronto ad ampio spettro tra tale metodo e una metodologia di prova a più ricevitori basata sull'analisi nel dominio frequenza-numero d'onda. Quest'ultima consente una notevole riduzione dei tempi di acquisizione ed interpretazione e presenta il vantaggio di essere meno sensibile all'influenza del rumore di fondo.

Il confronto viene effettuato utilizzando simulazioni numeriche della prova e dati sperimentali riguardanti differenti profili di rigidezza, riportando le differenze in termini di curva di dispersione sperimentale.

Infine il profilo di rigidezza corrispondente all'inversione dei dati relativi alle onde superficiali viene confrontato con i risultati di prove sismiche in foro, evidenziando l'affidabilità dei risultati ottenuti.